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CORRIGENDUM TO "EMBEDDING GRAPHS INTO COLORED GRAPHS"

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There is a technical mistake in the proof of Theorem 12 of [1]. The following is a correct exposition of the idea.

Theorem 12. *It is consistent that there exists a bipartite graph on* ω, ω_1 *vertices,* X, *such that* $Y \rightarrowtail (X)^2_2$ *holds for no* Y.

Proof. Let V be a model of ZFC + CH. Extend it by adding a Cohen real, i.e. by $P = \{p: \mathrm{Dom}(p) < \omega,\ \mathrm{Rng}(p) \subseteq \{0, 1\}\}$. Let $G: \omega \to \{0, 1\}$ be the generic function, $p_n = G|n$. Let $f_\alpha: \omega \to \omega$, $\alpha < \omega_1$ be a scale in V. For $n < \omega$, $\omega < \alpha < \omega_1$ let $\{n, \alpha\} \in E(X)$ iff $G(f_\alpha(n)) = 1$. Assume that in V^P, Y is a graph on λ, and 1 forces this fact. For $\{\alpha, \beta\} \in E(Y)$ let

$$n(\alpha, \beta) = \min\{n < \omega: p_n \Vdash \{\alpha, \beta\} \in E(Y)\}$$

and put $f(\alpha, \beta) = G(n(\alpha, \beta))$.

Notice that if for $p \in P$, $p \neq \varnothing$, p^- denotes the condition $p|(\mathrm{Dom}(p) - 1)$ and $p \Vdash \{\alpha, \beta\} \in E(Y)$, $p^- \nVdash \{\alpha, \beta\} \in E(Y)$ then $p \Vdash n(\alpha, \beta) = \mathrm{Dom}(p)$.

We claim that $f: E(Y) \to 2$ establishes that X does not embed into Y in either color. Assume indirectly that $g: \omega \to \lambda$, $h: \omega_1 - \omega \to \lambda$ embed X into Y in the ith color. Considering that $|P| = \omega$, there are a condition $p \in G$, a set $S \in [\omega_1]^{\omega_1} \cap V$, and a sequence $\{x_\alpha: \alpha \in S\} \in V$ such that p forces all the properties of g, h listed above, and $p \Vdash h(\alpha) = x_\alpha$ for $\alpha \in S$. For $k < \omega$ let $q_k \in P$ be such that $q_k \leq p$ and $q_k \Vdash g(k) = y_k$. Put $l(k) = \mathrm{Dom}(q_k)$. Since the f_α form a scale there is an $\alpha \in S$ such that $l(k) < f_\alpha(k)$ for some $k < \omega$. Let $r \leq q_k$ be such that $r(f_\alpha(k)) = 1$. Then $r \Vdash \{k, \alpha\} \in E(X)$ hence $r \Vdash \{y_k, x_\alpha\} \in E(Y)$. On the other hand $r^- \nVdash \{y_k, x_\alpha\} \in E(Y)$. This is true since r^- has an extension r' with $r'(f_\alpha(k)) = 0$, and then $r' \Vdash g(k) = y_k \wedge h(\alpha) = x_\alpha \wedge \{y_k, x_\alpha\} \notin E(Y)$. Then $p \geq r \Vdash n(y_k, x_\alpha) = \mathrm{Dom}(r)$ and $r \nVdash G(\mathrm{Dom}(r)) = i$, a contradiction.

References

1. A. Hajnal and P. Komjáth, *Embedding graphs into colored graphs*, Trans. Amer. Math. Soc. **307** (1988), 395–409.

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